BOUNDING NEGATIVE ENERGY WITH QUANTUM INFORMATION, CAUSALITY AND A LITTLE BIT OF CHAOS

based on arXiv: **1706.09432** with:

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NEGATIVE ENERGY DENSITY

- Globally energy in relativistic QFT is positive (above vacuum), but local energy density can be negative due to quantum fluctuations
- How negative?
- Why is negative bad?

some local negative energy density

Answer 1 (gravity) - you can use it to build time machines when coupling the QFT to gravity

Answer 2 (QFT) - violates micro-causality
- violates quantum information bounds

THE QUANTUM NULL ENERGY CONDITION (QNEC)

- Classical field theory, Null Energy Condition (NEC):

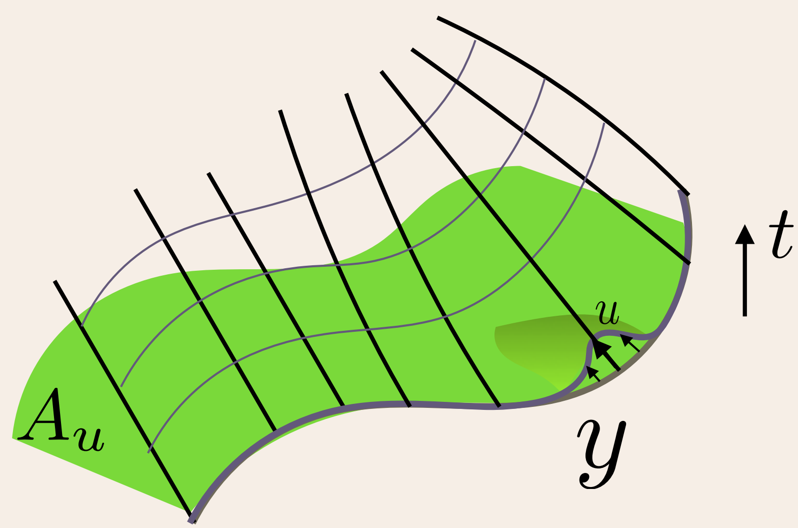
$$T_{uu}(y) \sim (\partial_u \phi)^2 \geq 0 \qquad u = \text{light-like coordinate}$$

- Quantum mechanics: violated by quantum fluctuations, but:

$$\langle T_{uu}(y) \rangle_\psi \geq \frac{\hbar}{2\pi \delta a} \frac{d^2 S_{EE}(A_u)}{du^2}$$

Bousso, Fisher, (Koeller), Leichenauer, Wall `15

Proof: free fields



A GENERAL PROOF OF THE QNEC?

- Links many different areas of study:

Quantum information

bulk reconstruction

Entanglement Hamiltonians

Entanglement entropy

chaos bound

ANEC

micro causality and the CFT bootstrap

defect CFTs

CHAOS

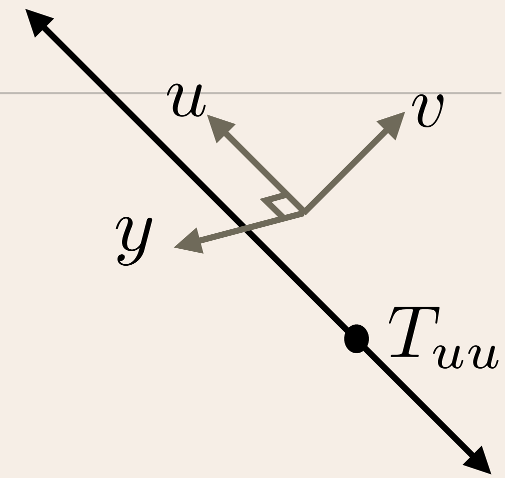
emergence of gravity spacetime from QFT via AdS/CFT??

THINGS TO KEEP IN MIND

- The QNEC is a conjectured property of all QFTs (no gravity)
- We will work in flat space, although should extend to QFTs in curved space
- We will work with general QFTs with an interacting UV fixed point and d>2
- We start our story with the ANEC …

TWO PATHS TO THE ANEC

THE ANEC



- The averaged NEC: $\langle\, \widehat{\mathcal{A}}_u(y)\, \rangle_\psi \geq 0$

$$\widehat{\mathcal{A}}_u \equiv \int_{-\infty}^{\infty} du' T_{uu}(u', v' = 0, y)$$

Null momentum: $P_u = \int dy \widehat{\mathcal{A}}_u(y)$

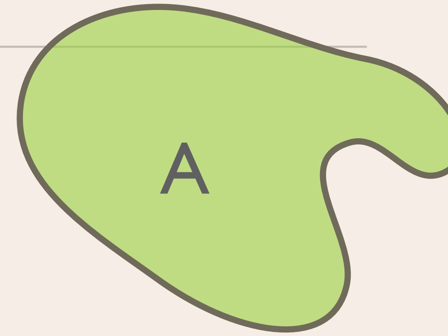
- Predicted from GR (e.g. wormholes not traversable)
- Non trivial in Minkowski space ~ Hofman-Maldacena bounds

In d=3+1 CFTs: $\frac{31}{18} \geq \frac{a}{c} \geq \frac{1}{3}$

METHOD I: ENTANGLEMENT HAMILTONIANS

TF, Leigh, Parrikar, Wang, `16

$$\mathcal{H}_{tot} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}} \qquad \rho_A = \mathrm{Tr}_{\bar{A}} \left|\psi\right\rangle \left\langle\psi\right|$$



$$\rho_A = e^{-2\pi H_A}$$

- "Full" Entanglement Hamiltonian - better behaved in QFT:

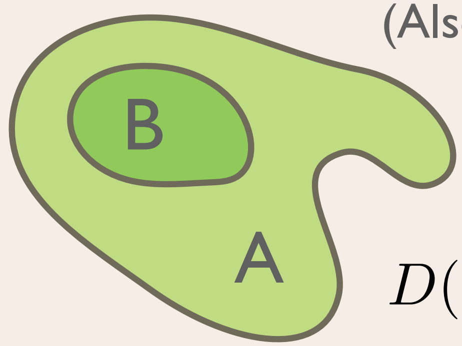
$$K_A = H_A \otimes 1_{\bar{A}} - 1_A \otimes H_{\bar{A}}$$

(Also called modular Hamiltonian)

- Inclusion property:

$$B \subset A$$

$$K_A - K_B \geq 0$$

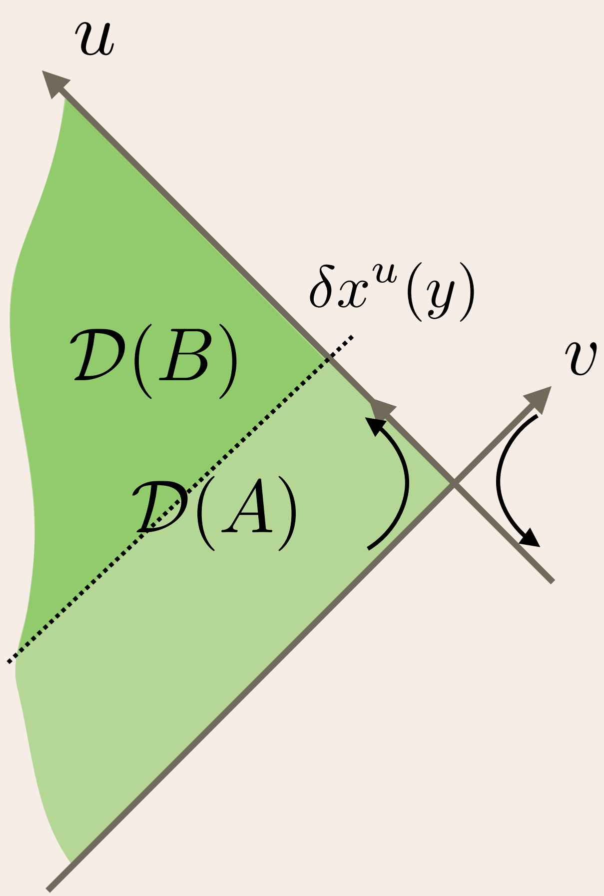


$$D(\rho_A || \sigma_A) \geq D(\rho_B || \sigma_B)$$

Proof: relative entropy monotonicity

MODULAR HAMILTONIANS

Relativistic QFT - inclusion at level of causal domains



- vacuum state: $|\psi\rangle \to |0\rangle$
- A: half space (Rindler) cut

  $K_A^0 =$ Boost operator

- B: small null deformation thereof

$$K_A^0 - K_B^0 = \int dy \delta x^u \widehat{A}_u(y)$$

Uniform cuts = null momentum op.

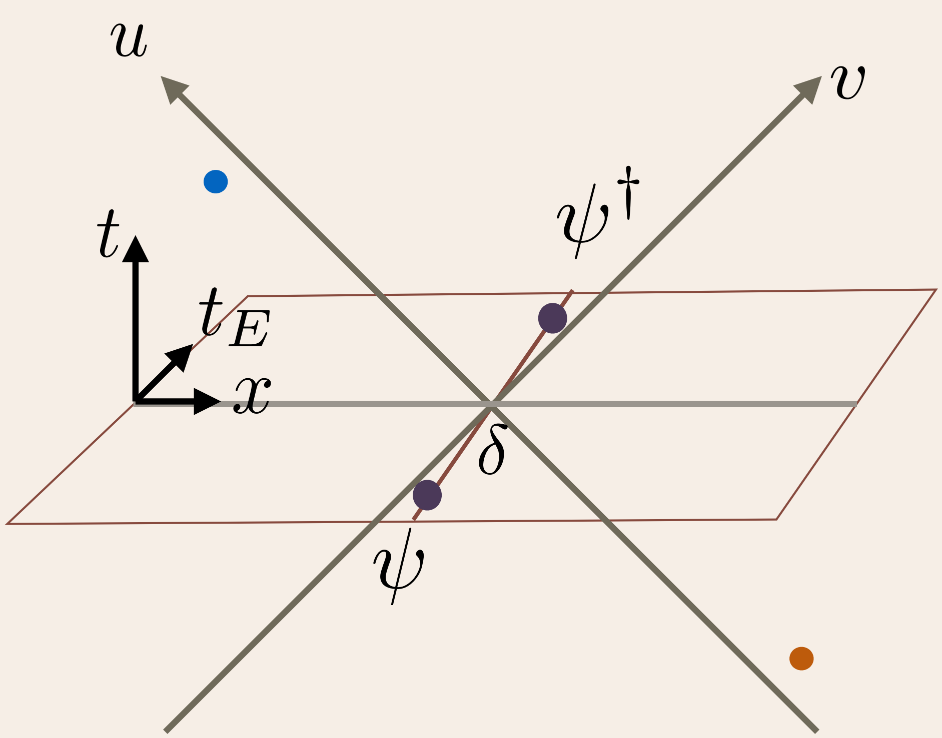
Non-uniform cuts = ANEC

$\longrightarrow$ $\widehat{A}_u(y) \geq 0$

METHOD II: CAUSALITY

Hartman, Kundu, Tajdini `16

Space like separated operators: $[\mathcal{O}, \mathcal{O}] = 0$



True for any state:

$$\langle\psi| [\mathcal{O}, \mathcal{O}] |\psi\rangle = 0$$

Operator quenches:

$$|\psi\rangle \equiv \psi(t = -i\delta) |0\rangle$$

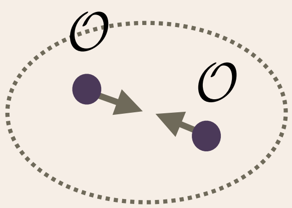
4 point function! (Bootstrap)

$$f = Z^{-1} \langle 0| \psi \mathcal{O} \mathcal{O} \psi |0\rangle$$

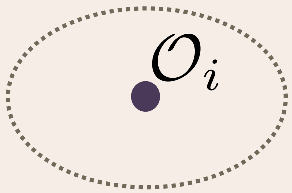
COMPUTABLE IN LIGHTCONE LIMIT

$$f \propto \langle 0|\, \psi \mathcal{O}\mathcal{O} \psi \,|0\rangle$$

Operator Product Expansion:

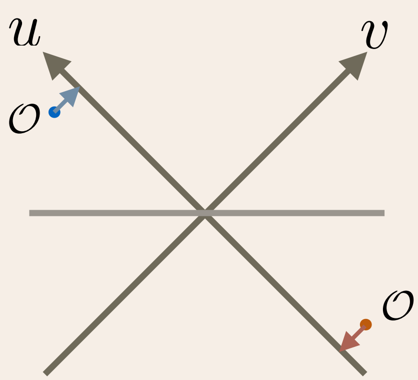


$$= \sum_i$$

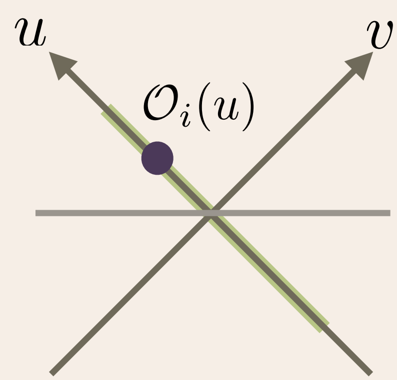


Dominated by lowest dimension operator

Light-cone Operator Product Expansion:



$$= \sum_i \int du g(u)$$



Dominated by lowest twist operator = dimension - spin

$$\mathcal{O}_i = T_{uu}$$

COMPUTABLE IN LIGHTCONE LIMIT

$$f \propto \langle 0 | \psi \mathcal{O} \mathcal{O} \psi | 0 \rangle$$

- Indeed the ANEC operator dominates in this limit: $\beta = 2\pi$

$$f(s) = 1 - \kappa e^{2\pi s/\beta} \langle \mathcal{A}_u \rangle_\psi + \dots$$

Hartman, Kundu, Tajdini `16

- Small correction in light-cone limit:

$$\kappa \propto v^{d-2} \qquad v \to 0$$

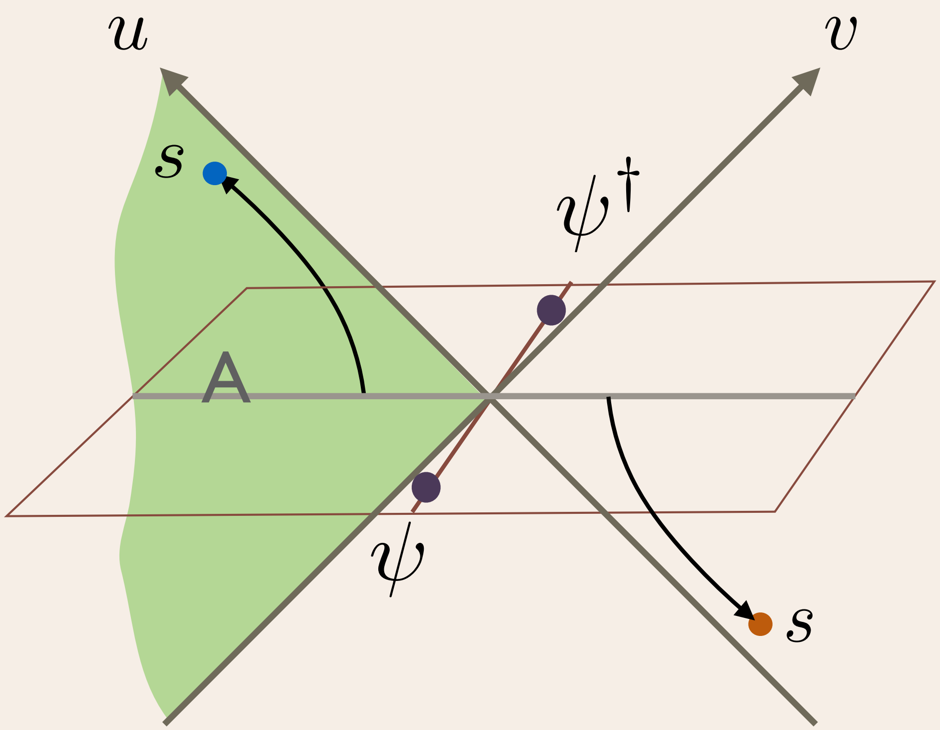
- Same as chaos bound story for OTOC …

Maldacena, Shenker, Stanford (MSS)

- What is this time, s?

SECRETLY AN OUT OF TIME ORDER CORRELATION

$f \propto \langle 0| \psi \mathcal{O} \mathcal{O} \psi |0\rangle$



Vacuum entangled like a thermofield double state:

Light cone limit achieved with boost after sending $\mathcal{O}\mathcal{O}$ together

Defines:

$$f(s) = \langle 0| \psi^\dagger \mathcal{O}(s) \mathcal{O}(s) \psi |0\rangle$$

But reduced density matrix for A looks thermal w.r.t. boost operator!

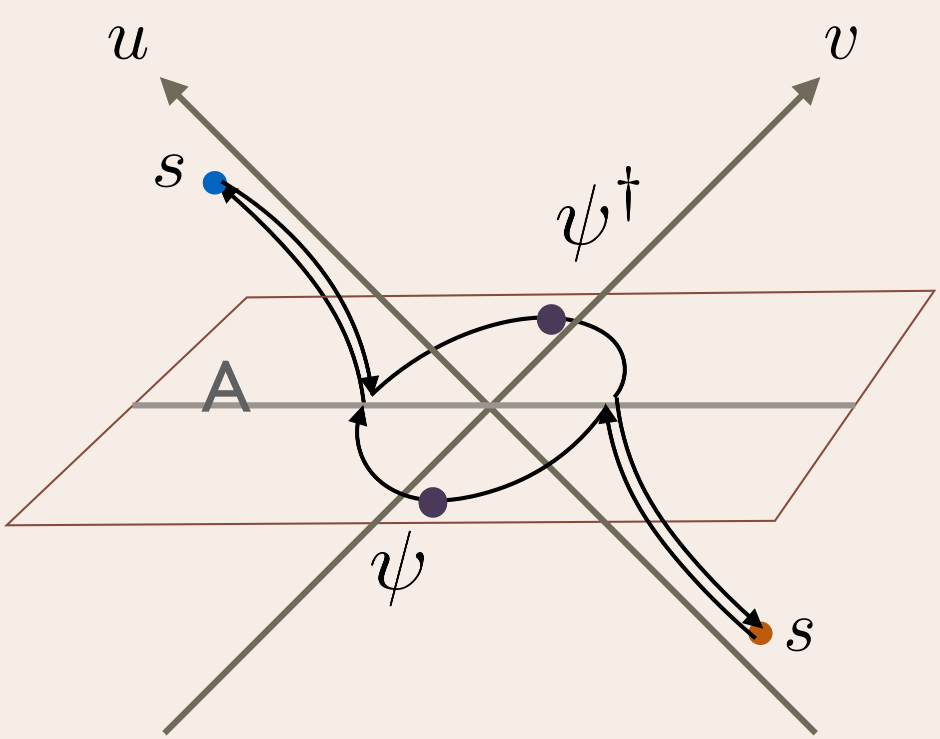
$$\beta = 2\pi$$

$$|0\rangle = \sum_\alpha e^{-\beta E_\alpha/2} |\alpha\rangle_A |\alpha\rangle_{\bar{A}}$$

$$\longrightarrow \rho_A = e^{-2\pi H_A}$$

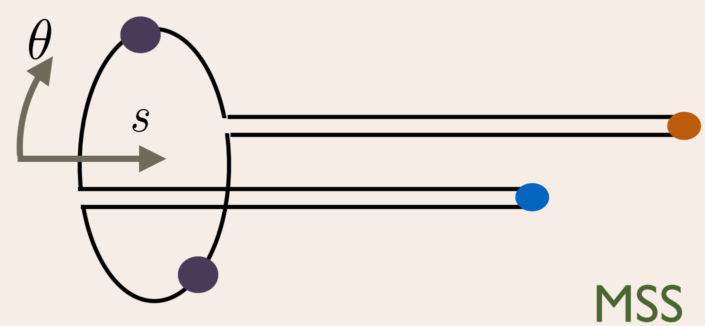
SECRETLY AN OUT OF TIME ORDER CORRELATION

$$f \propto \langle 0|\, \psi \mathcal{O} \mathcal{O} \psi \,|0\rangle$$



Boosts in imaginary time
become rotations around `thermal circle'

Reduce to thermal correlator
(Schwinger-Keldysh contour)



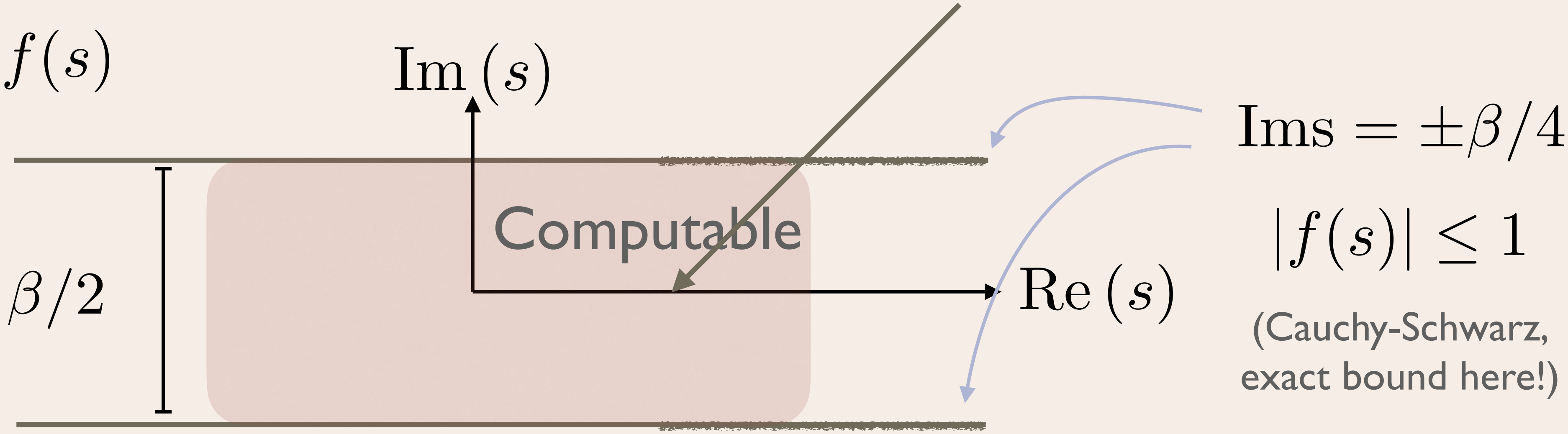
Can be used to extract
Commutator squared:

$$s \to s + i\beta/4$$

$$f(s) \ni \mathrm{Tr}\left(e^{-\beta H_A/2}\,[\psi, \mathcal{O}(s)]\right)^2$$

BOUND ON CHAOS:

- Properties of chaos function: $f(s) = 1 - \epsilon e^{\lambda_L s} + \ldots$



$\Rightarrow \quad \lambda_L \leq 2\pi/\beta \qquad \epsilon \geq 0$

Hartman, Kundu, Tajdini `16

- Here: $f(s) = 1 - \kappa e^{2\pi s/\beta} \langle \mathcal{A}_u \rangle_\psi + \ldots$

Saturates the Chaos bound, but ANEC: $\langle \mathcal{A}_u \rangle_\psi \geq 0$

BOUND ON CHAOS:

- Note that we did not need a large N limit - small parameter determined by kinematics of light-cone limit
- All (interacting) theories have same Lyapunov exponent in this limit - so this kind of Chaos not very discriminating
- Chaotic behavior: $\psi(-i\epsilon)\left|0\right\rangle$

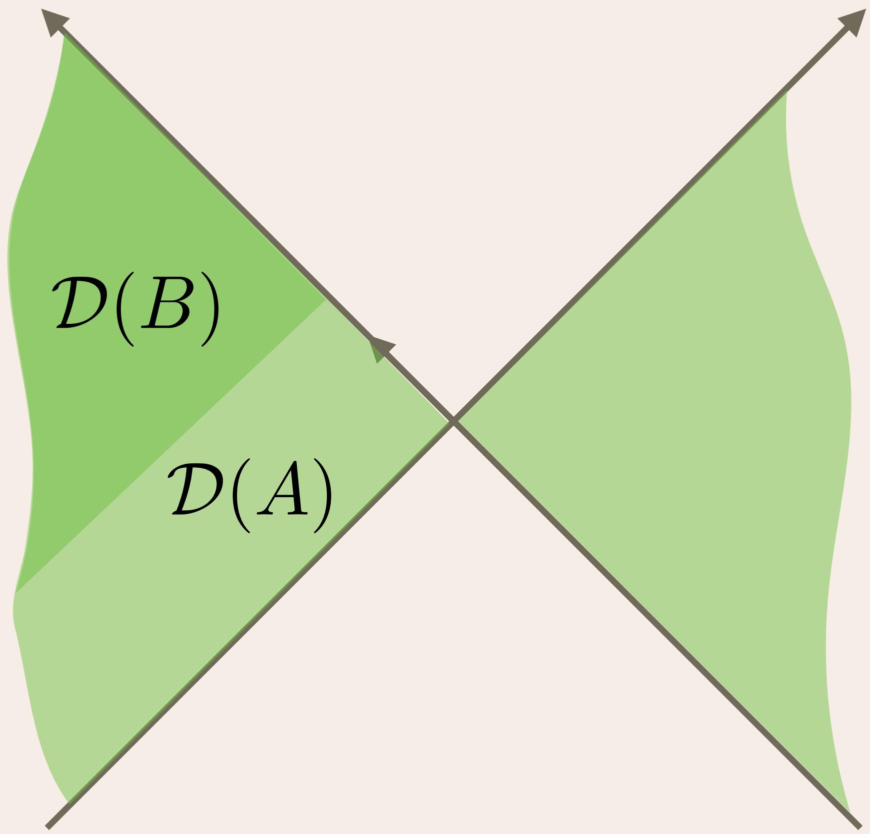
  Disrupts local correlations/entanglement between A and $\bar{A}$ that exist in vacuum. Results in exponential decay of boosted correlator:

$$\left\langle\psi\right|\mathcal{O}_A(s)\mathcal{O}_{\bar{A}}(s)\left|\psi\right\rangle$$

HOW ARE THESE RELATED??

Quick answer: I have no idea

COMBINING THEM



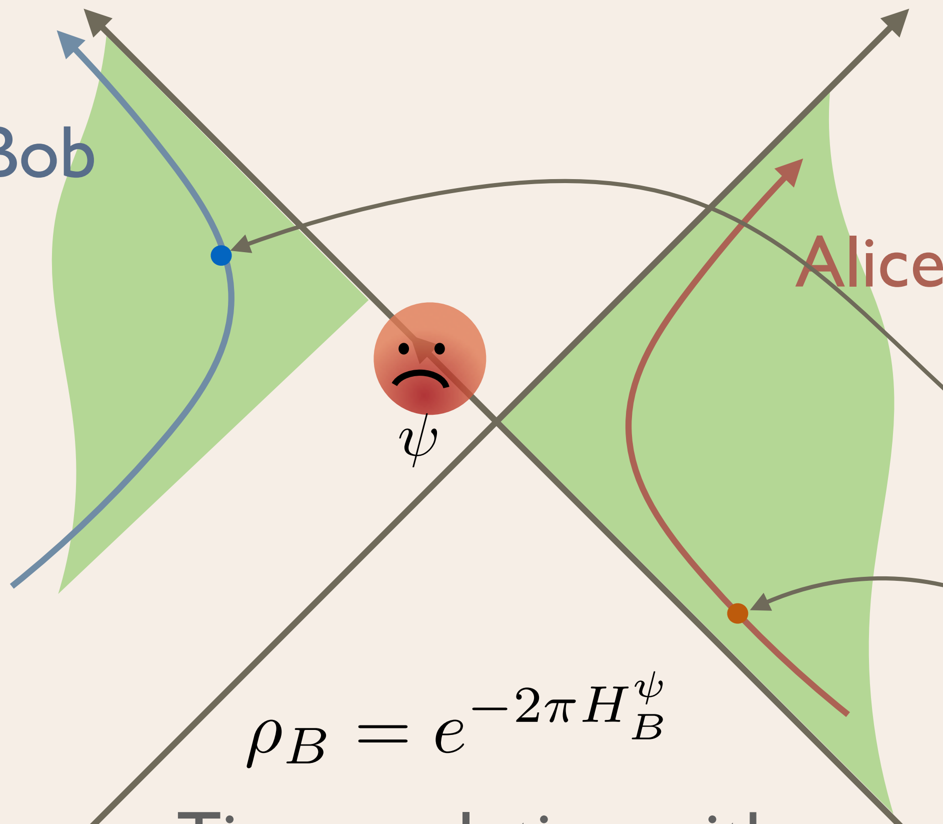
Inclusion property:

$$\mathcal{D}(B) \subset \mathcal{D}(A)$$

Causally disconnected:

$$\left[\mathcal{D}(B), \mathcal{D}(\bar{A})\right] = 0$$

COMBINING THEM



$$\rho_B = e^{-2\pi H_B^{\psi}}$$

Time evolution with entanglement Hamiltonian!

$$[M_B, M_A] = 0$$

Now allow for non-local operators

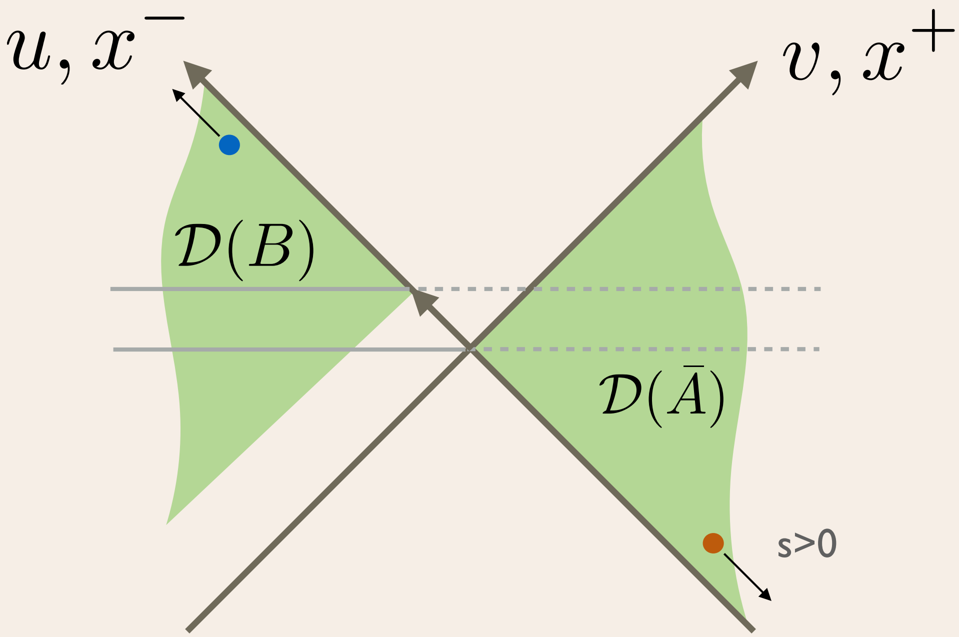
Interesting ops:

$$M_B(s) = \rho_B^{is} \mathcal{O}_B \rho_B^{-is}$$

$$M_A(s) = \rho_{\bar{A}}^{is} \mathcal{O}_{\bar{A}} \rho_{\bar{A}}^{-is}$$

Note ρ_B reduced density matrix for: $|\psi\rangle \langle\psi|$

SETUP



Our new Chaos function is:

$$f(s) \propto \langle\psi| \, M_B(s) M_A(s) \, |\psi\rangle$$

Using results from Algebraic QFT, one can show that $f(s)$ has all the desired properties

Challenge: computing it!! New operators are non-local and messy objects .... light cone limit!

RESULT

- working in same ANEC light-cone limit for the operator insertions:

$$f(s) = 1 - \kappa e^{2\pi s/\beta} Q_u + \ldots \qquad \kappa \ll 1$$

$$\mathcal{Q}_u = \int_{\partial A}^{\partial B} du' T_{uu}(u', v' = 0, y) + \left(\frac{\delta S_{EE}(\rho_A)}{\delta X^u(y)} - \frac{\delta S_{EE}(\rho_B)}{\delta X^u(y)} \right)$$

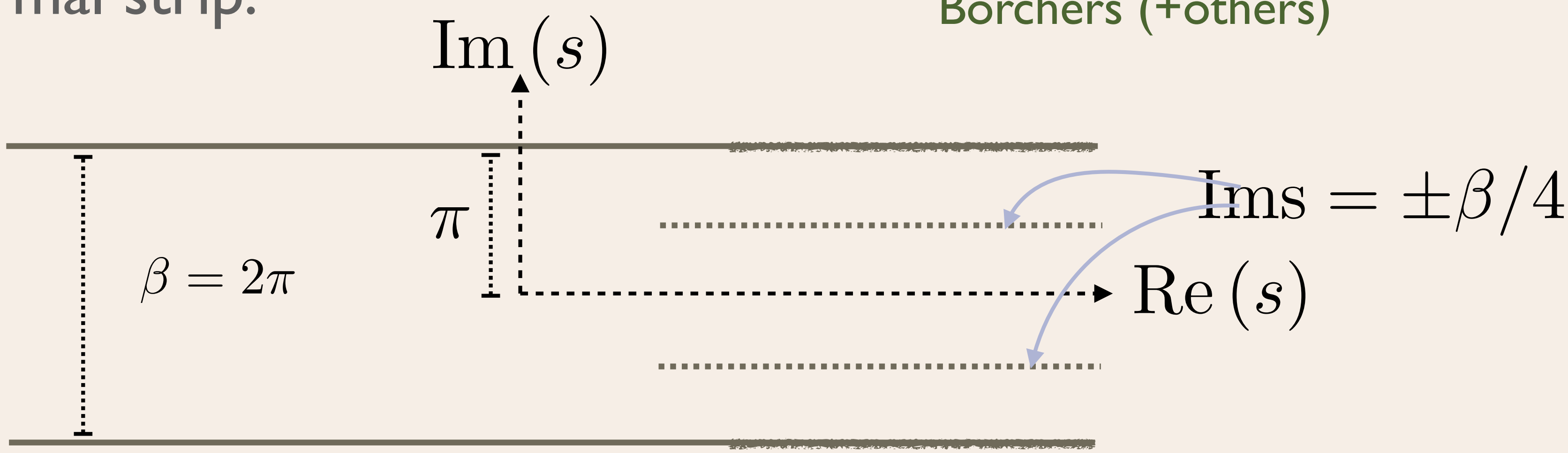
- Computed using a defect OPE in the replica trick

Evolving with ψ entanglement Hamiltonian ~ to leading order is the vacuum boost plus computable corrections …

PROPERTIES:

- **Causality:** entanglement time evolution ~ thermal time for non-equilibrium states ~ analog KMS condition ~ analyticity in the thermal strip:

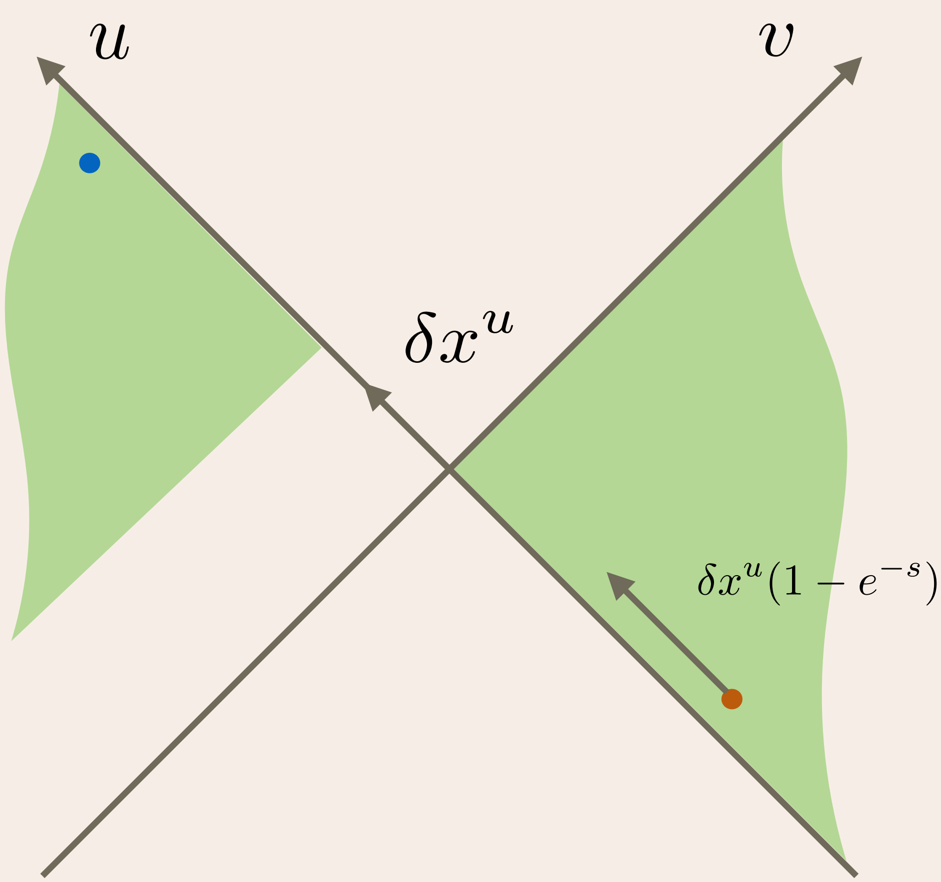
Borchers (+others)



- Cauchy-Schwarz bound also applies, so by same logic arrive at QNEC: $Q_u \geq 0$

Balakrishnan, TF, Khandker, Wang

SOME INTUITION - CAUSALITY

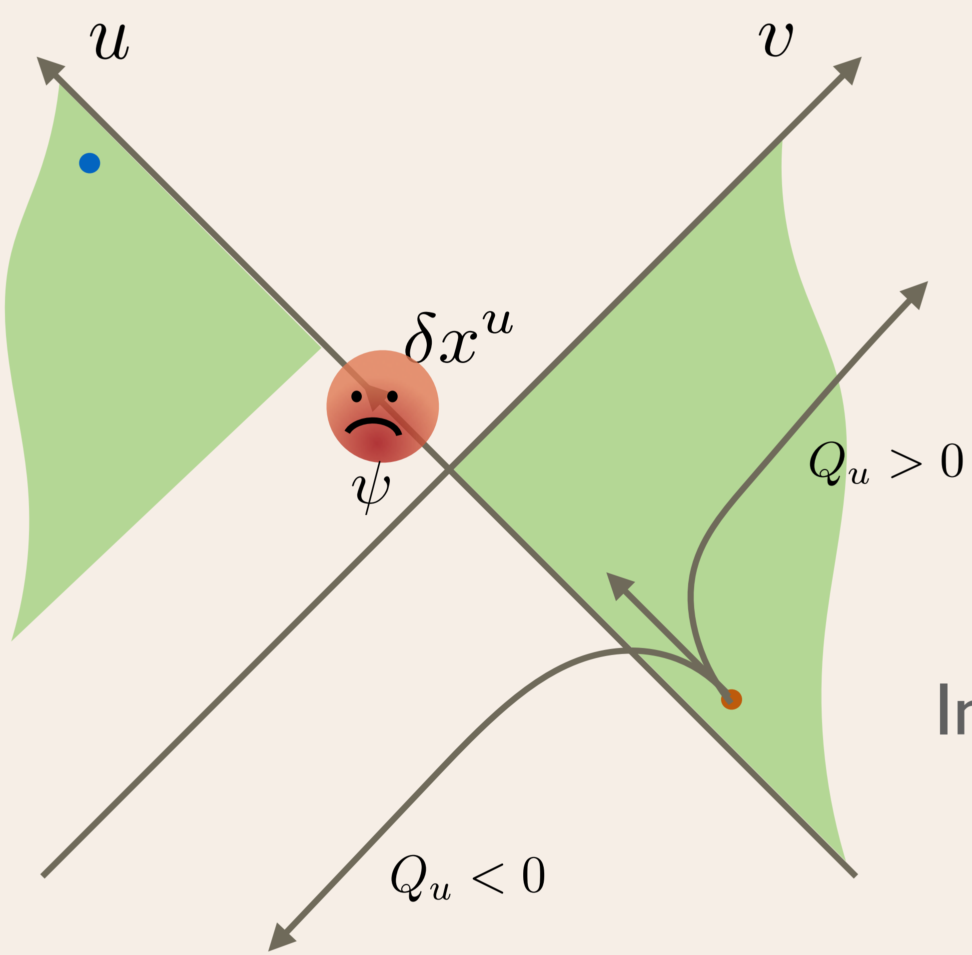


Vacuum flow:

$$\langle 0 | \, {M_B}^0(s) {M_A}^0(s) \, | 0 \rangle$$

Composition of two null separated boosts results in a mild null translation …

SOME INTUITION - CAUSALITY



Vacuum flow:

$$\langle 0| \, {M_B}^0(s) {M_A}^0(s) \, |0\rangle$$

Composition of two null separated boosts results in a mild null translation …

In excited state boosts slightly misalign and error amplified **bigly**.

NO TIME FOR …

- AdS/CFT interpretation (this was the main motivation for this calculation - and also gave us the idea to use entanglement time evolution)
- Relation to bulk reconstruction (see above)
- The actual calculation (we used several new techniques for dealing with entanglement in QFT)
- Higher spin version of the QNEC

QUESTIONS FOR AUDIENCE

- Are there useful bounds one can derive using similar methods in CM context? (i.e. no relativistic symmetry)
- Entanglement spectrum/Hamiltonians studied in topological phases. What happens if you time evolve with such Hamiltonians?
- What are the implications for a bound on the **acceleration** of entanglement entropy? (i.e. the QNEC.)